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Delhi School of Economics
Department of Economics

Entrance Examination for M.A. Economics
OPTION B
June 30, 2007

Time: 3 hours **Maximum Marks: 100**

General Instructions.

- Check that this Examination Question booklet has pages 1 – 7 and you have been given a blank Answer booklet for writing your answers. Do not start writing until instructed to do so by the invigilator.
- This examination has 2 parts. Part I carries 20 marks of compulsory questions. In Part II, you must answer 4 out of 7 questions, each carrying 20 marks.
- Fill in your Name and Roll Number on the small slip attached to the Answer booklet. Do **not** write this information anywhere else in this booklet.
- The candidature of anyone found engaging in illegal examination practices will be cancelled.

PART I

This compulsory part consists of 10 multiple choice questions. For each of them, indicating the correct answer in your Answer booklet will give you 2 marks, indicating a wrong answer will deduct 2/3rd of a mark, and not giving any answer will give a zero mark.

A. If the statement "There exists a legislature and a party such that every legislator in that party pays taxes." is false, then which of the following statements must be true?

(a) In every legislature and every party, all legislators do not pay taxes.

(b) There exists a legislature such that every legislator in every party does not pay taxes.

(c) In every legislature, there exists a party and a legislator in that party who does not pay taxes.

(d) In every legislature and every party, there exists a legislator who does not pay taxes.

B. If $f : \Re^n \to \Re$ is twice differentiable, concave and homogeneous of degree 1, then the Hessian matrix of f is

(a) negative definite
(b) positive definite
(c) singular
(d) non-singular

C. If $f : \Re_+ \to \Re$ is defined by $f(x) = \int_x^{x^2} e^t \, dt$, then the derivative of f at $x > 0$ is

(a) $2xe^{x^2} - e^x$
(b) $2xe^{x^2} + e^x$
(c) $(2x - 1)e^{x^2}$
(d) $(2x - 1)e^x$

D. The sequence (x_n), where $x_n = (-1)^n(1 + n^{-1})$ and $n = 1, 2, \ldots,$

(a) converges to 1
(b) converges to -1
(c) converges to 1 and -1
(d) converges to neither 1 nor -1

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E. The set $\cap_{n=1}^{\infty}(-1-n^{-1}, 1+n^{-1})$ is identical to
(a) $(-1, 1]$
(b) $[-1, 1)$
(c) $(-1, 1)$
(d) $[-1, 1]$

F. For $x, y \in \Re^n$, let $d(x, y) = \max\{|x_i - y_i| \mid i = 1, \ldots, n\}$. Which of the following relations holds for all $x, y, z \in \Re^n$?
(a) $d(x, z) = d(x, y) + d(y, z)$
(b) $d(x, z) > d(x, y) + d(y, z)$
(c) $d(x, z) \geq d(x, y) + d(y, z)$
(d) $d(x, z) \leq d(x, y) + d(y, z)$

G. Given sets X, Y, let $X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$ and let $X - Y = \{x \in X \mid x \notin Y\}$. Which of the following formulae is generally correct for sets A, B and C?
(a) $(A - B) \times C = (B \times C) - (A \times C)$
(b) $(A - B) \times C = (A \times C) - (B \times C)$
(c) $(A - B) \times C = (A \times C) - B$
(d) $(A - B) \times C = A - (B \times C)$

H. The average travel time to a distant city is w hours by train or z hours by bus. A man cannot decide whether to take the train or the bus, so he tosses a coin. What is his expected travel time?
(a) $2(w + z)$
(b) $2(w + z)/(w - z)$
(c) $(w + z + 2)/2$
(d)None of the above

I. The coefficient of x^2 in the polynomial
$(1 - x)(1 + 2x)(1 - 3x) \ldots (1 + 14x)(1 - 15x)$ is
(a) -121
(b) -191
(c) -255
(d) -291

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J. A continuous random variable has the probability density function $f(x) = 1/3$, for x between -1 and $+2$, and 0 elsewhere. Its mean, variance and median are:

(a) $(1, 3/4, 1/2)$
(b)$(1/2, 3/4, 1/2)$
(c) $(1/2, 1, 1)$
(d) $(1/2, 1, 1/2)$

PART II

Answer any 4 of the 7 questions below. Each question is worth 20 marks.

1. Compute the required probabilities in each of the following cases:

   (a) Consider two events A and B such that $Pr(A) = \frac{1}{3}$ and $Pr(B) = \frac{1}{2}$. Determine the value of $Pr(BA^c)$ for each of the following conditions:

   i. A and B are disjoint
   ii. $A \subset B$
   iii. $Pr(AB) = \frac{1}{8}$.

   (b) If four dice are rolled, what is the probability that each of the four numbers that appear will be different?

   (c) A deck of thoroughly shuffled cards is dealt out to 4 players, so that each of them receive 13 cards. What is the probability that each player will receive exactly one ace?

   (d) Consider an experiment where the sample space contains four outcomes $\{s_1, s_2, s_3, s_4\}$, and suppose that the probability of each outcome is $\frac{1}{4}$. Let the three events A, B and C be defined as follows:

   $$A = \{s_1, s_2\},\ B = \{s_1, s_3,\} \text{ and } C = \{s_1, s_4\}$$

   Are these events independent? Are they *pairwise* independent?

(e) In the city of Delhi, 50% of eligible voters most prefer the Congress Party, 30% most prefer the BJP, and the remaining 20% most prefer other parties. It turns out that 82% of those that most prefer the Congress, 65% of those that most prefer the BJP, and half of the third group voted in the last election. If a citizen (eligible to vote) is picked at random, and it is found that he did not vote in the last election, what is the probability that the Congress is his/her most preferred party?

(f) Suppose that a fire can occur at any one of five points along a road. These points are given by {-3,-1,0,1,2} and the probabilities associated with these five points are given by {.2,.1, .1, .4, .2}. At what point along the road should a fire engine wait to minimize the expected value of the square of the distance that it must travel to the next fire?

2. (a) Suppose that the random variables $X_1, ... X_n$ form a random sample from a Bernoulli distribution for which the parameter θ is unknown ($0 \leq \theta \leq 1$). Consider the observed values $x_1, \ldots x_n$, where each x_i is either 0 or 1:

   i. Write down the likelihood function for the sample.

   ii. Use this to compute the maximum likelihood estimate of θ.

   (b) Suppose a point in the xy plane is chosen at random from the interior of a circle for which the equation is $x^2 + y^2 = 1$; and suppose that the probability that the point will belong to any region inside the circle is proportional to the area of that region. Let Z denote a random variable representing the distance from the center of the circle to the point. Find and sketch the distribution function of Z.

3. Imagine that you are to sample from a uniform distribution on the interval $(0, \theta)$, where the value of θ is unknown. You would like to test the hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 2$. Denote by α, the probability of rejecting the null hypothesis when it is true, and β the probability of not rejecting it when it is false.

   (a) For a sample of size $n = 1$, find a test procedure for which $\alpha = 0$

and β is minimized, given this value of α. What is the value of β in this case?

(b) For a sample $X_1, X_2, \dots X_n$ of size n, what is the minimum value of β that can be attained among all procedures for which $\alpha = 0$?

4. Consider the vector space $\Re^n$, with the Euclidean norm $\| \|$. A point x is said to be a *closure point* of a set P ($P \subseteq \Re^n$) if for all $\epsilon > 0$, there exists a point $p \in P$ such that $\|x - p\| < \epsilon$. The set of all closure points of P is called the *closure of* P and denoted $\overline{P}$.

A point x is said to be an *interior point* of a set P if there is an open ball centred at x that is contained entirely in P. The set of all interior points of P is denoted $int(P)$.

A set P is said to be *convex* if for every (x, y, t) such that $x \in P$, $y \in P$ and $t \in [0, 1]$, it is true that $tx + (1 - t)y \in P$.

(a) Let $P \subseteq \Re^n$. Show that $\overline{\overline{P}} = \overline{P}$. (i.e., the closure of $\overline{P}$ equals the closure of P).

(b) Let $P \subseteq \Re^n$ be a convex set. Show that int(P) is a convex set.

5. (a) Let $[a, b]$ and $[c, d]$ be nonempty closed intervals in $\Re$. Let $f : [a, b] \times [c, d] \rightarrow \Re$ be a continuous function defined on the rectangle $[a, b] \times [c, d]$. Define a function $g : [a, b] \rightarrow \Re$ as follows. For every $x \in [a, b]$, let $g(x) = max\{f(x, y) | y \in [c, d]\}$. Show that g is a continuous function.

(b) An *ordered pair* (a, b) of two objects a and b is defined to be the set $\{\{a\}, \{a, b\}\}$. Show that for any two ordered pairs (a, b) and (a', b'), $(a, b) = (a', b')$ if, and only if, $a = a'$ and $b = b'$.

6. (a) Show that for a sequence $\{x_m\}$ of real numbers to be a *Cauchy sequence*, it is necessary, but not sufficient that $|x_{m+1} - x_m|$ converges to zero.

(b) Let X be an $n \times k$ real matrix, with $k < n$. Let $R(X)$ be the range space of X, i.e., the space of all linear combinations of the columns of X. Let $R^{\perp}(X) = \{w \in \Re^n | \ w.z = 0, \ for\ all\ z \in R(X)\}$. (In terms of coordinates, $w = (w_1, ..., w_n)$, $z = (z_1, ..., z_n)$, so $w.z$ is

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defined to equal $\sum_{i=1}^{n} w_i z_i$). Show that if $Rank(X) = k$, then the subspace $R^{\perp}(X)$ has dimension (n-k).

7. (a) Show that the eigenvalues of the partitioned matrix

$$\left(\begin{array}{c|c} A & C \\ \hline 0 & B \end{array}\right)$$

(where A and B are square submatrices and 0 is a submatrix of zeros), are the eigenvalues of A and B.

(b) Suppose $A \subseteq \Re^2$, and let $B \subseteq \Re$ be defined by $B = \{x \in \Re | (x, y) \in A, \ for\ some\ y \in \Re\}$
Prove, or provide a counterexample to, the statement: "If A is a closed set in $\Re^2$, then B is a closed set in $\Re$".